Christ Church Grammar School

2021
TEST 4

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	2 minutes
Working time for this section:	15 minutes
Marks available:	15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

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2. Write your answers in this Question/Answer Booklet using a blue or black pen. Do not use erasable or gel pens.

3. Answer all questions.

4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 1 **(6 marks)**

Solve for x in the following equations, using exact values where necessary:

(a) $5^x = 10^{2-x}$ (3 marks)

(b) $(\ln x)^2 - 10\ln(x) + 24 = 0$ (3 marks)

See next page

Question 2 **(3 marks)**

Determine the equation of the tangent to the curve $y = \ln(\sin x)$ at the point where $x = \frac{\pi}{4}$.

See next page

Question 3 **(3 marks)**

Consider $y = \ln((2e + x)^3)$. Showing use of the Increments formula, approximate the small change in y, when x changes from $6e$ to $7e$.

See next page

Question 4 **(3 marks)**

Determine the following:

(a) $\frac{d}{dx}(\ln(x^2 - 5x))$ (1 mark)

(b) $\int \frac{5x^2}{x^3+10} dx$ (2 marks)

End of questions

Christ Church Grammar School

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TEST 4

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section

Reading time before commencing work:	3 minutes
Working time for this section:	30 minutes
Marks available:	30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

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See next page

Question 5 **(6 marks)**

A continuous random variable X has pdf:

$$f(x) = \begin{cases} 0.0228x^2 + 0.01 \quad for \quad 0 \leq x \leq 5 \\ 0 \; for \; all \; other \; values \; of \; x. \end{cases}$$

(a) Determine $P(1 < X < 3)$. (1 mark)

(b) Determine Var(X). (3 marks)

(c) Determine the cumulative distribution function for the random variable X. (2 marks)

See next page

Question 6 **(6 marks)**

The time that Finn arrives at school for ice hockey training is uniformly distributed between 6:58 am and 7:08 am. Finn is considered late if he arrives after 7:00 am.

(a) Determine the probability that Finn arrives

(i) at exactly 7:00 am. (1 mark)

(ii) between 7:02 am and 7:08 am. (1 mark)

(iii) after 7:05 am, given that he is late. (2 marks)

(b) During a particular term, Finn attends ice hockey training on 30 occasions. Determine the probability that he is late on at least 20 of these occasions. (2 marks)

See next page

Question 7 **(4 marks)**

A continuous random variable X has pdf:

$$f(x) = ax^2 \ for \ 0 \leq x \leq b$$

If $P(X \leq 1) = \frac{1}{8}$, then determine the value of a and b.

See next page

Question 8 **(7 marks)**

Staples sells rulers with lengths normally distributed with a mean of 100 cm and a standard deviation of 0.95 cm.

(a) Determine the proportion of rulers that are between 98 cm and 101cm. (1 mark)

(b) If 250 rulers were purchased, how many would you expect to be between 98 cm and 101 cm? (2 marks)

(c) Determine the smallest length of the largest 5% of rulers to 3 decimal places. (2 marks)

(d) Staples advertises that the rulers are 1 metre in length. What will the mean of the distribution need to be if only 2% of rulers are to be below 1 metre in length? Assume the standard deviation remains at 0.95 cm. (2 marks)

See next page

Question 9 **(7 marks)**

The pH (pouvior hydrogene – hydrogen power) of a solution is a measure of its hydrogen ion concentration. It is calculated using the formula: $pH = -log_{10}H^+$, where H^+ is the concentration of (H^+) ions in the solution (moles/litre). Pure water at $22°C$ has a concentration of 1×10^{-7} moles/litre.

(a) Calculate the pH of water at $22°C$. (2 marks)

(b) Calculate the concentration of hydrogen ions in a solution with pH of 8.7. (2 marks)

(c) Solution A has a pH of 9 whereas solution B has a pH of 3. Calculate the ratio of hydrogen ions in solution B to that in solution A in the form $x : 1$. (3 marks)

End of questions

Additional working space

Question number: _____

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MATHEMATICS METHODS Year 12

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Calculator-free

Your name - SOLUTIONS -

Teacher's name _____

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Question 1 **(6 marks)**

Solve for x in the following equations, using exact values where necessary:

(a) $5^x = 10^{2-x}$ (3 marks)

$\log 5^x = \log 10^{2-x}$ ✓ [log both sides]

$x \log 5 = (2-x) \log 10$ → $\log 10 = 1$

✓ [uses log laws]

$x \log 5 + x = 2$

$x(\log 5 + 1) = 2$

OR $\frac{\ln 100}{\ln 50}$ $\boxed{x = \frac{2}{\log 5 + 1}}$ ✓ [isolates x]

(b) $(\ln x)^2 - 10(\ln x) + 24 = 0$ (3 marks)

let $p = \ln x$

$\therefore p^2 - 10p + 24 = 0$ ✓ [forms quad]

$(p-6)(p-4) = 0$

$p = 6$ OR $p = 4$ ✓ [solves quadratic]

$\ln x = 6$ OR $\ln x = 4$

$\therefore \boxed{x = e^6}$ OR $\boxed{x = e^4}$ ✓ [Both sol^{n}s]

6

Question 2 **(3 marks)**

Determine the equation of the tangent to the curve $y = \ln(sinx)$ at the point where $x = \frac{\pi}{4}$.

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}$ $\frac{1}{sinx} \cdot \cos x$

$\therefore \frac{\cos \frac{\pi}{4}}{\sin \frac{\pi}{4}} \Rightarrow \frac{\sqrt{2}/2}{\sqrt{2}/2}$

$m = 1$ ✓ [gradient]

$\therefore y = 1x + c \Rightarrow$ Sub $\left(\frac{\pi}{4}, \ln \frac{\sqrt{2}}{2}\right)$

$\ln \frac{\sqrt{2}}{2} = \frac{\pi}{4} + c$

$c = \ln \frac{\sqrt{2}}{2} - \frac{\pi}{4}$ ✓ [c-value]

$\therefore$ Equ $y = x + \ln \frac{\sqrt{2}}{2} - \frac{\pi}{4}$ ✓ [Equ]

OR $y = x - \frac{1}{2}\ln(2) - \frac{\pi}{4}$

OR $y = x - \ln\sqrt{2} - \frac{\pi}{4}$

3

Question 3 **(3 marks)**

Consider $y = \ln(2e + x)^3$. Showing use of the Increments formula, approximate the small change in y, when x changes from $6e$ to $7e$.

$\frac{dy}{dx} \approx \frac{\delta y}{\delta x}$

$\delta x = 7e - 6e$

$\delta x = e$

$\frac{dy}{dx} = \frac{1}{(2e+x)^3} \times \frac{3(2e+x)^2 \times 1}{1}$

$\frac{\delta y}{\delta x} = \frac{3}{(2e+x)}$ ✓ [Diff]

$\delta y = \frac{3}{(2e+x)} \times e$

$\delta y = \frac{3e}{2e+6e}$ ✓ [subs $x = 6e$, $\delta x = e$]

$\delta y \approx \frac{3}{8}$ ✓ [ANSW]

3

See next page

Question 4 **(3 marks)**

Determine the following:

(a) $\frac{d}{dx}(ln\,(x^2 - 5x))$ (1 mark)

$= \frac{2x-5}{x^2-5x}$ ✓

(b) $\int \frac{5x^2}{x^3+10} dx$ Assume $x^3+10 > 10$ (2 marks)

$= \frac{5}{3}\int \frac{3x^2}{x^3+10} dx$

$= \frac{5}{3} \ln|x^3+10| + C$

✓ ✓ [No need for abs value]

-1 if no +C

End of questions

3

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MATHEMATICS METHODS Year 12

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Teacher's name _____

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Question 5 **(6 marks)**

A continuous random variable X has pdf:

$$f(x) = \begin{cases} 0.0228x^2 + 0.01 \quad for \quad 0 \leq x \leq 5 \\ 0 \; for \; all \; other \; values \; of \; x. \end{cases}$$

(a) Determine $P(1 < X < 3)$ (1 mark)

$\int_1^3 0.0228x^2 + 0.01 \, dx$ = 0.2176 ✓ $\left(\frac{136}{625}\right)$

(b) Determine $Var(X)$ (3 marks)

$E(X) = \int_0^5 x \times f(x) \, dx$ = 3.6875 $\left(\frac{59}{16}\right)$ ✓

$VAR(X) = \int_0^5 f(x) \times \underbrace{[x - 59/16]^2}_{[x-\mu]^2} \, dx$ ✓ [correct formula]

= 1.069 $\left(\frac{821}{768}\right)$

(c) Determine the cumulative distribution function for the random variable X. (2 marks)

0.0076

$$P(X \leq x) = \begin{cases} 0 \quad for \quad x < 0 \\ \frac{19x^3}{2500} + \frac{x}{100} \quad for \quad 0 \leq x \leq 5 \quad ✓ \\ 1 \quad for \quad x > 5 \quad ✓ \text{ [must have 1]} \end{cases}$$

$\int_0^5 f(x) \, dx$

6

Question 6 **(6 marks)**

The time that Finn arrives at school for ice hockey training is uniformly distributed between 6:58am and 7:08am. Finn is considered late if he arrives after 7:00am.

(a) Determine the probability that Finn arrives:

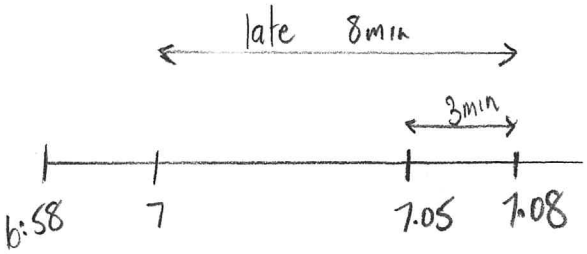
(i) at exactly 7:00am. (1 mark)

$\underline{0}$ ✓

(ii) between 7:02am and 7:08am. (1 mark)

$\frac{6}{10}$ ✓

(iii) after 7:05am, given that he is late. (2 marks)



$\Rightarrow \frac{P(X > 7.05)}{P(X > 7)}$ ✓ (NUMER)

$= \frac{3}{8}$ ✓ (ANSW)

(b) During a term, Finn attends ice hockey training on 30 occasions. Determine the probability that he is late on at least 20 of these occasions. (2 marks)

$Y \sim B(30, 0.8)$ ✓ [BINOMIAL]

$P(X \geq 20) = 0.9744$ ✓ [ANSW]

6

Question 7 **(4 marks)**

A continuous random variable X has pdf:

$$f(x) = ax^2 \ for \ 0 \leq x \leq b$$

If $P(X \leq 1) = \frac{1}{8}$, determine the value of a and b.

$\int_0^1 ax^2 \, dx = 1/8$ ✓ (OR (CPAD)) [$\int$ statement]

$\left[\frac{ax^3}{3}\right] = \frac{1}{8}$

$a = 3/8$ ✓ [value]

$\int_0^b 3/8 \, x^2 \, dx = 1$ ✓ [$\int$ statement]

$b = 2$ ✓ [value]

4

Question 8 **(7 marks)**

Staples sells rulers with lengths normally distributed with a mean of 100 cm and a standard deviation of 0.95cm.

(a) Determine the proportion of rulers that are between 98 cm and 101cm. (1 mark)

$\mu = 100$

$\sigma = 0.95$

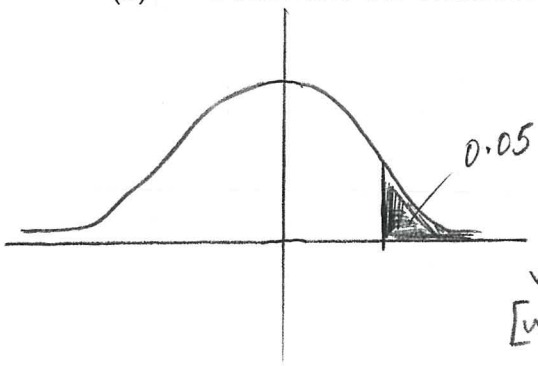
$P(98 < X < 101) = 0.8361$ ✓

(b) If 250 rulers were purchased, how many would you expect to be between 98 cm and 101 cm? (2 marks)

$250 \times 0.8361 = 209.025$ ✓

ie ~ 209 rulers ✓ [Int. value]

(c) Determine the smallest length of the largest 5% of rulers. (to 3dp) (2 marks)

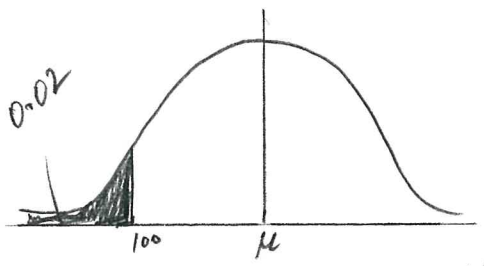


✓ [working]

solve (Norm cdf $(x, \infty, 0.95, 100) = 0.05$)

$x = 101.563$ cm ✓

(d) Staples advertises that the rulers are 1 metre in length. What will the mean of the distribution need to be if only 2% of rulers are to be below 1 metre in length? Assume the standard deviation remains at 0.95cm (2 marks)



✓ [working]

solve (Norm cdf $(-\infty, 100, 0.95, z) = 0.02$)

$\mu = 101.95$ cm ✓

OR $z = -2.053749$

$z = \frac{100 - \mu}{0.95}$

solve for μ

See next page

Question 9 **(7 marks)**

The pH (pouvior hydrogene – hydrogen power) of a solution is a measure of its hydrogen ion concentration. It is calculated using the formula: $pH = -log_{10}H^+$, where H^+ is the concentration of (H^+) ions in the solution (moles/litre). Pure water at 22°C has a concentration of 1×10^{-7} moles/litre.

(a) Calculate the pH of water at 22°C. (2 marks)

$ph = -\log_{10} H^+$

$ph = -\log_{10} 1\times10^{-7}$ ✓ [subs 1×10^{-7}]

$= 7$ ✓ [ANSW]

(b) Calculate the concentration of hydrogen ions in a solution with pH of 8.7. (2 marks)

$8.7 = -\log_{10} H^+$ ✓ [subs 8.7]

$10^{-8.7} \approx 2\times10^{-9}$ moles/L ✓ [ANSW]

1.99526×10^{-9}

(c) Solution A has a pH of 9 whereas solution B has a pH of 3. Calculate the ratio of hydrogen ions in solution B to that in solution A in the form $x : 1$. (3 marks)

$3 = -\log_{10} H^+ \Rightarrow H^+ = 10^{-3}$ (B)

$9 = -\log_{10} H^+ \rightarrow H^+ = 10^{-9}$ (A)

✓ obtains both H^+

B : A

$10^{-3} : 10^{-9}$ ✓ [ratio]

1,000,000 : 1 ✓ [ratio $x:1$]

End of questions